Gravitational Radiation

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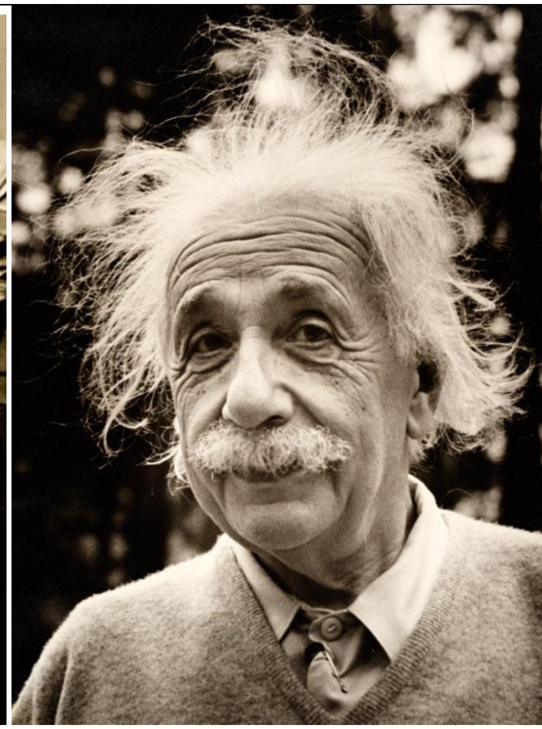
Abstract

Gravitational waves are ripples in the fabled fabric of the Universe that propagate as waves outward from their source at the speed of light. They were proposed by Henri Poincaré in 1905 and subsequently predicted in 1916 by Albert Einstein on the basis of his general theory of relativity. In this article, we deduce the basic equations which predict the gravitational waves from orbiting binaries, such as the two masses orbiting each other in highly circular orbit about their center of mass didactically following the simplest way maintaining, however, the necessary mathematical rigor.

Gravitational waves are 'ripples' in space-time, generated by accelerated masses that propagate as waves outward from their source at the speed of light. They were proposed by **Henri Poincaré** (French mathematician, theoretical physicist, engineer, and philosopher of science) in 1905 and subsequently predicted in 1916 by **Albert Einstein** on the basis of his general theory of relativity.

Gravitational waves were first directly detected by the Laser Interferometer Gravitational-Wave Observatory (**LIGO**) in 2015. Gravitational wave is to gravity what light is to electromagnetism. It is the transmission of variations in the gravitational field as waves. Predicted by Einstein's theory of general relativity, the waves transport energy known as gravitational radiation. Two objects orbiting each other in highly elliptical orbit or circular orbit about their center of mass comprises binary system. This system loses mass by emitting gravitational wave (**ripple in the geometry of space and time**) whose frequency $\upsilon = \frac{E}{h} \ll$ frequency of electromagnetic radiation

and this is associated with an in-spiral or decrease in orbit. Suppose that the two masses are m_1 and m_2, and they are separated by a distance "r" orbiting each other in highly circular orbit about their center of mass. The rate of loss of energy from the binary system through gravitational radiation is given by:

$$P = -\frac{dE}{dt} = \frac{32\,G^4\,(m_1 m_2)^2}{5c^5}\;\frac{(m_1 + m_2)}{r^5}$$

where $G = 6.674 \times 10^{-11}\ m^3\,kg^{-1}s^{-2}$ is the Newtonian gravitational constant and $c = 3 \times 10^8\ ms^{-1}$ is the speed of light in vacuum. Gravitational radiation robs the energy of orbiting masses. As the energy of the orbiting masses reduces, the distance between the masses decreases, and they orbit more rapidly. More generally, the rate of decrease of distance between the masses with time is given by:

$$v = -\frac{dr}{dt} = \frac{64\,G^3\,(m_1 m_2)}{5c^5}\;\frac{(m_1 + m_2)}{r^3}$$

$$2P = -2\,\frac{dE}{dt} = \frac{64\,G^4\,(m_1 m_2)^2}{5c^5}\;\frac{(m_1 + m_2)}{r^5}$$

$$2P = -\frac{dr}{dt} \times \frac{Gm_1 m_2}{r^2}$$

$$2P = v \times F_G$$

$$\boxed{F_G = \frac{2P}{v}}$$

where F_G is the force of gravitation between the two masses orbiting each other in highly circular orbit about their center of mass.

$$F_G = \frac{2P}{v} = 2\,\frac{dE}{dr}$$

The loss of energy through gravitational radiation could eventually drop the mass m_1 into the mass m_2. The lifetime of distance "r" between the masses orbiting each other in highly circular orbit about their center of mass is given by:

$$t_{life} = \frac{5c^5r^4}{256G^3(m_1m_2)(m_1+m_2)}$$

$$\frac{4t_{life}}{r} = \frac{5c^5r^3}{64G^3(m_1m_2)(m_1+m_2)} = \frac{1}{v}$$

$$\boxed{v = \frac{r}{4t_{life}}}$$

$$F_G = \frac{2P}{v} = \frac{8Pt_{life}}{r}$$

$$P = \frac{F_G r}{8t_{life}} = -\frac{U_G}{8t_{life}}$$

where U_G is the energy is associated with the state of separation between two orbiting masses that attract each other by the gravitational force.

Two orbiting masses are moving at a common orbital angular frequency (in radians/sec) given by:

$$\omega = \sqrt{\frac{G\,(m_1+m_2)}{r^3}}$$

$$\boxed{\omega^2 = \frac{G\,(m_1+m_2)}{r^3}}$$

$$t_{life} = \frac{5}{256} \times \frac{c^5}{G} \times \frac{r}{Gm_1m_2} \times \frac{r^3}{G\,(m_1+m_2)}$$

$$t_{life} = \frac{5}{256} \times \frac{\text{Planck power}}{\omega^2} \times \frac{r}{Gm_1m_2}$$

$$t_{life} = \frac{5}{256} \times \frac{\text{Planck power}}{\omega^2} \times \frac{1}{F_G r}$$

$$\boxed{U_G = -\frac{5 \times \text{Planck power}}{256\omega^2 t_{life}}}$$

$$P = -\frac{U_G}{8t_{life}} = \frac{5 \times \text{Planck power}}{2048\omega^2 t_{life}^2}$$

The gravitational wave signal was observed by LIGO detectors in Hanford and in Livingston on **14 September 2015**. An exact analysis of the gravitational wave signal based on the Albert Einsteinian theory of general relativity showed that it came from two merging stellar black holes with 29 and 36 solar masses, which merged 1.3 billion light years from Earth. Before the merger, the total mass of both black holes was 36 + 29 solar masses = 65 solar masses. After the merger, the mass of resultant black hole was 62 solar masses.

What happened to three solar masses?

It was turned into the energy transported by the emitted gravitational waves. Using Albert Einstein's equation $E = mc^2$, where E is the energy transported by the emitted gravitational waves, m is the missing mass (**3 solar masses**) and c is the speed of light, we can estimate the energy released as gravitational waves:

$$E = (3 \times 2 \times 10^{30}\ \text{kg}) \times (3 \times 10^{8}\ \text{m/s})^{2}$$

$$E = 5.4 \times 10^{47}\text{J}$$

This is roughly 10^{21} more energy than the complete electromagnetic radiation emitted by our sun.

$$\upsilon = \frac{E}{h} = \frac{5.4 \times 10^{47}}{6.626 \times 10^{-34}} = 8.14 \times 10^{80}\text{s}^{-1}$$

Reduced mass:

$$\mu = \frac{m_1 m_2}{(m_1 + m_2)}$$

$$\omega = \sqrt{\frac{G m_1 m_2}{\mu r^3}}$$

The moment of inertia of the binary system:

$$I = \mu r^2$$

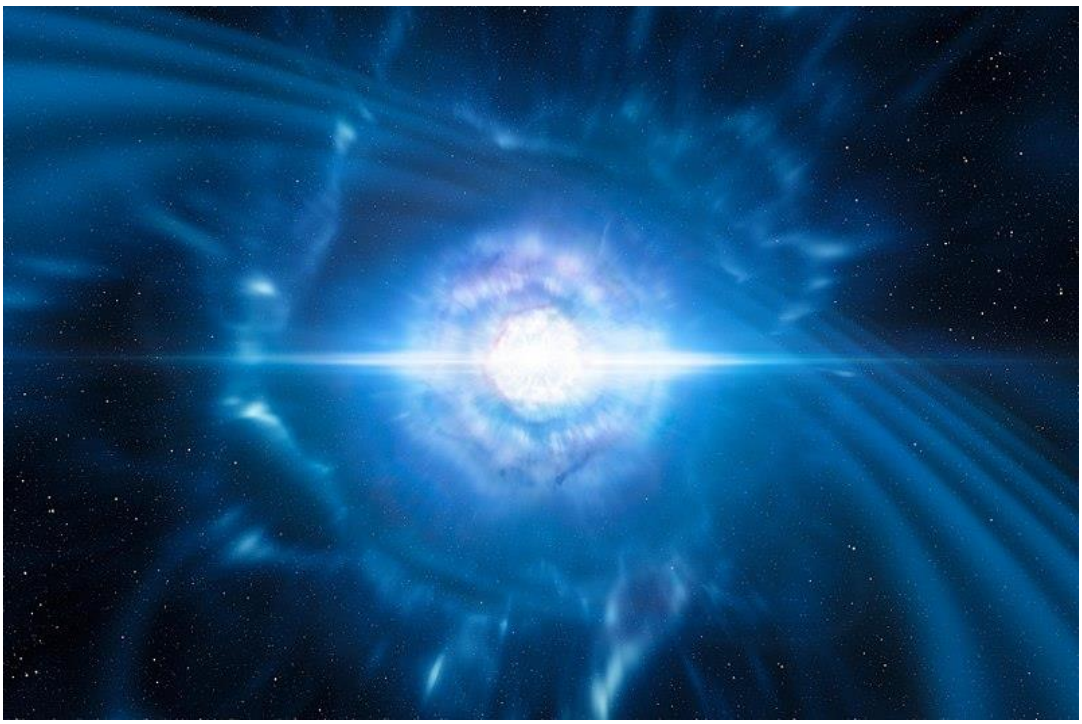
$$\omega = \sqrt{\frac{F_G r}{I}} = \sqrt{\frac{8Pt_{life}}{I}}$$

"Newton's law of gravitation. That's all you need (with a spot of calculus to crunch the numbers) to work out how the Earth will orbit the Sun or how an apple will fall if you let it go at a certain height. The only trouble is that Newton had no idea how this gravity thing worked. His model was simply: 'There is an attraction between bits of stuff, and let's not bother about why."

— Brian Clegg

$$I \times \omega^2 = \frac{Gm_1 m_2}{r}$$

$$\boxed{P = -\frac{U_G}{8t_{life}} = \frac{I\omega^2}{8t_{life}}}$$



Artist's impression of merging neutron stars, a source of gravitational waves

$$P = \frac{32}{5} \times \frac{G}{c^5} \times \frac{G^2(m_1 m_2)^2}{r^2} \times \frac{G\,(m_1+m_2)}{r^3}$$

$$\boxed{P = \frac{32I^2\omega^6}{5 \times \text{Planck power}}}$$

$$\frac{I\omega^2}{8t_{life}} = \frac{32I^2\omega^6}{5 \times \text{Planck power}}$$

$$\boxed{t_{life} = \frac{5 \times \text{Planck power}}{256I\omega^4}}$$

$$v = \frac{r}{4t_{life}} = r \times \frac{64I\omega^4}{5 \times \text{Planck power}}$$

Since:

$$I = \mu r^2$$

Therefore:

$$\boxed{v = \frac{64\omega^4 I^{3/2}}{5 \times \sqrt{\mu} \times \text{Planck power}}}$$

Albert Einstein theorized that smaller masses travel toward larger masses, not because they are "attracted" by a mysterious force called gravity, but because the smaller objects travel through space that is warped by the larger object.

The amplitude of gravitational waves gets smaller with the distance to the source.

Sources of Gravitational waves

- Exploding stars
- Rotating Neutron stars

- **Gravity** → Curvature of 4-dimensional (3 space + 1 time) space-time fabric produced by matter.
- **Gravitational-waves** → Ripples on 4-dimensional space-time produced by accelerated matter.

The orbital energy of binary system:

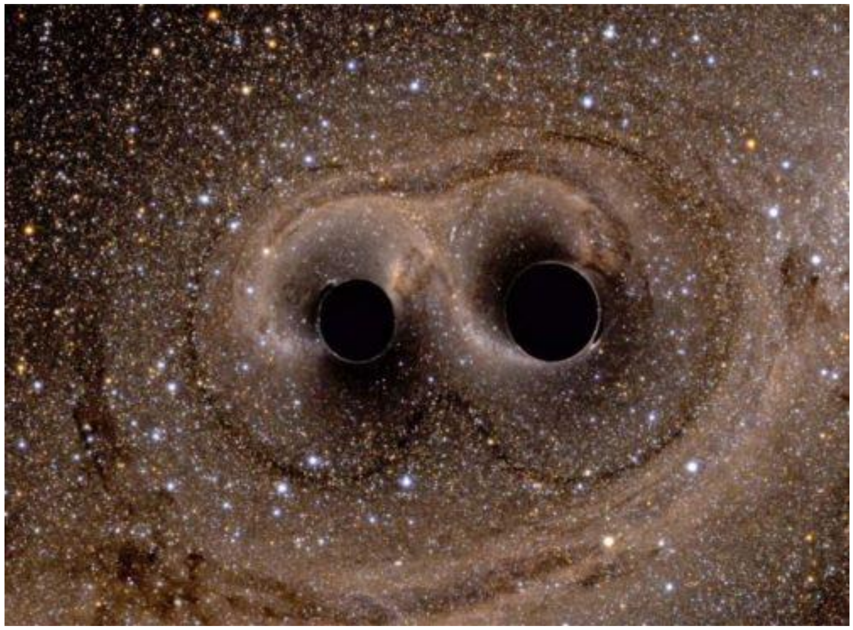
$$E = \frac{\mu v_{orb}^2}{2} - \frac{Gm_1 m_2}{r} = \frac{I v_{orb}^2}{2r^2} - 8Pt_{life}$$

v_{orb} is the relative orbital velocity of two orbiting masses

Angular momentum:

$$L = I\omega$$

$$P = \frac{I\omega^2}{8t_{life}} = \frac{L\omega}{8t_{life}}$$



An artist's illustration of two black holes coalescing. As they collide, the two black holes produce gravitational waves that ripple through the fabric of space-time.

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